Calcul des déformations des fils élastiques

Fils élastiques en arc de cercle - Force concentrée parallèle à l'axe du fil

Flexion et torsion

Fil rond en cuivre

$d := 0.6 \cdot mm \qquad S := \pi \cdot \frac{d^2}{4} \qquad E := 1.1 \cdot 10^5 \cdot N \cdot mm^{-2} \qquad G := \frac{E}{2.6} \qquad \rho := 8.9 \cdot 10^3 \cdot kg \cdot m^{-3}$

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$J_t := J_{t\_circ}(d) \qquad I_{22} := I_{f\_circ}(d) \qquad I_{33} := I_{22}$

$W_t := W_{t\_circ}(d) \qquad W'_t := W_t \qquad W_{f2} := W_{f\_circ}(d) \qquad W_{f3} := W_{f2}$

Caractéristiques de l'arc de cercle $\quad R := 21.5 \cdot mm \qquad \psi_{AB} := 75 \cdot deg$

Forces extérieures en bout d'arc

$F_x := 0 \cdot N \qquad F_y := 0 \cdot N \qquad F_z := 0.01 \cdot N \qquad C_x := 0 \cdot N \cdot mm \qquad C_y := 0 \cdot N \cdot mm \qquad C_z := 0 \cdot N \cdot mm$

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - F&C.mcd(R)

Valeur de tests transitoires $\quad \alpha_m := 20 \cdot deg$

Torseur des forces de cohésion

$\mathbf{M_c}(\psi_{AB}, \alpha_m)^T = (0.134 \quad 0.146 \quad 0)\, N \cdot mm$

Sollicitations

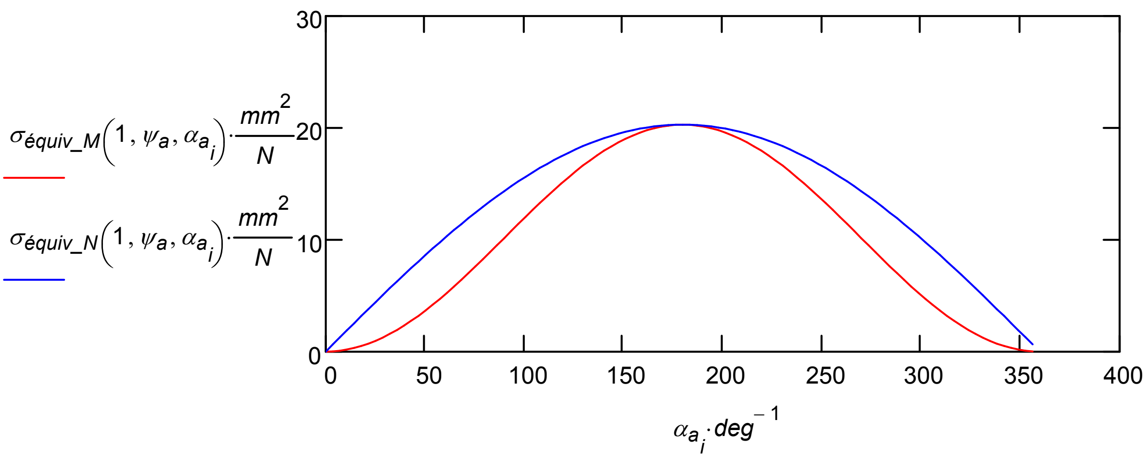
$\mathbf{e'_1}(\alpha_m)^T = (-0.342 \quad 0.94 \quad 0) \qquad \mathbf{e'_2}(\alpha_m)^T = (-0.94 \quad -0.342 \quad 0) \qquad \mathbf{e'_3}(\alpha_m)^T = (0 \quad 0 \quad 1)$

Moment de torsion $\quad M_t(\psi_{AB}, \alpha_m) = 0.092\, N \cdot mm$

Moments de flexion $\quad M_{f2}(\psi_{AB}, \alpha_m) = -0.176\, N \cdot mm \qquad M_{f3}(\psi_{AB}, \alpha_m) = 0\, N \cdot mm$

Contraintes

Cas d'un anneau fendu $\quad n := 101 \qquad i := 1 .. n-1 \qquad \psi_a := 360 \cdot deg \qquad \alpha_{a_i} := (i-1) \cdot \frac{\psi_a}{n-1}$



Calcul des déplacements par les intégrales de Mohr

Position du déplacement désiré $\alpha_M := 40 \cdot deg$

Calcul des déplacements linéiques

Déplacement dans la direction de Ox $\lambda := 0 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$

$\delta_x(\psi_{AB}, \alpha) := \delta_v(\psi_{AB}, \alpha, \lambda, \gamma)$ $\boxed{\delta_x(\psi_{AB}, \alpha_M) = 0\,mm}$

Déplacement dans la direction de Oy $\lambda := 90 \cdot deg$ $\gamma := 90 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv2}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$ $\delta_{fv3}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$

$\delta_y(\psi_{AB}, \alpha) := \delta_v(\psi_{AB}, \alpha, \lambda, \gamma)$ $\boxed{\delta_y(\psi_{AB}, \alpha_M) = 0\,mm}$

Déplacement dans la direction de Oz $\lambda := 0 \cdot deg$ $\gamma := 0 \cdot deg$ $|\mathbf{v}(\lambda, \gamma)| = 1$

$\delta_{tv}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 5.929 \times 10^{-3}\,mm$ $\delta_{fv2}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0.029\,mm$ $\delta_{fv3}(\psi_{AB}, \alpha_M, \lambda, \gamma) = 0\,mm$

$\delta_z(\psi_{AB}, \alpha) := \delta_v(\psi_{AB}, \alpha, \lambda, \gamma)$ $\boxed{\delta_z(\psi_{AB}, \alpha_M) = 0.035\,mm}$

Calcul des déplacements angulaires

Déplacement angulaire autour de Ox $\lambda_c := 0 \cdot deg$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = -0.04\,deg$ $\theta_{fcv2}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0.197\,deg$ $\theta_{fcv3}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$

$\theta_x(\psi_{AB}, \alpha) := \theta_{cv}(\psi_{AB}, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_x(\psi_{AB}, \alpha_M) = 0.157\,deg}$

Déplacement angulaire autour de Oy $\lambda_c := 90 \cdot deg$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0.142\,deg$ $\theta_{fcv2}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0.065\,deg$ $\theta_{fcv3}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$

$\theta_y(\psi_{AB}, \alpha) := \theta_{cv}(\psi_{AB}, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_y(\psi_{AB}, \alpha_M) = 0.208\,deg}$

Déplacement angulaire autour de Oz $\lambda_c := 0 \cdot deg$ $\gamma_c := 0 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv2}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$ $\theta_{fcv3}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$

$\theta_z(\psi_{AB}, \alpha) := \theta_{cv}(\psi_{AB}, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_z(\psi_{AB}, \alpha_M) = 0\,deg}$

Déplacement angulaire de flexion $\lambda_c := \alpha_M$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0.061\,deg$ $\theta_{fcv2}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0.193\,deg$ $\theta_{fcv3}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$

$\theta_f(\psi_{AB}, \alpha) := \theta_{cv}(\psi_{AB}, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_f(\psi_{AB}, \alpha_M) = 0.254\,deg}$

Déplacement angulaire de torsion $\lambda_c := \alpha_M + \frac{\pi}{2}$ $\gamma_c := 90 \cdot deg$ $|\mathbf{cv}(\lambda, \gamma)| = 1$

$\theta_{tcv}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0.135\,deg$ $\theta_{fcv2}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = -0.077\,deg$ $\theta_{fcv3}(\psi_{AB}, \alpha_M, \lambda_c, \gamma_c) = 0\,deg$

$\theta_t(\psi_{AB}, \alpha) := \theta_{cv}(\psi_{AB}, \alpha, \lambda_c, \gamma_c)$ $\boxed{\theta_t(\psi_{AB}, \alpha_M) = 0.058\,deg}$

Solution analytique

Moment de torsion $M_t(\psi_{AB},\alpha') := F_z \cdot R \cdot (1 - \sin(\psi_{AB}) \cdot \sin(\alpha') - \cos(\psi_{AB}) \cdot \cos(\alpha'))$

Moment fléchissant $M_{f2}(\psi_{AB},\alpha') := F_z \cdot R \cdot (\cos(\psi_{AB}) \cdot \sin(\alpha') - \sin(\psi_{AB}) \cdot \cos(\alpha'))$

Déplacement selon Oz $M_{tv}(\alpha,\alpha') := R \cdot (1 - \cos(\alpha - \alpha'))$ $\qquad M_{fv}(\alpha,\alpha') := -R \cdot \sin(\alpha - \alpha')$

$$\delta_{t3}(\psi_{AB},\alpha) := \frac{F_z \cdot R^3}{2 \cdot G \cdot J_t} \cdot \left[2 \cdot \alpha + \alpha \cdot \cos(\psi_{AB} - \alpha) - 2 \cdot (\sin(\psi_{AB}) + \sin(\alpha)) + 2 \cdot \sin(\psi_{AB} - \alpha) + \cos(\psi_{AB}) \cdot \sin(\alpha)\right]$$

$$\delta_{f3}(\psi_{AB},\alpha) := \frac{F_z \cdot R^3}{2 \cdot E \cdot I_{22}} \cdot (\alpha \cdot \cos(\psi_{AB} - \alpha) - \sin(\alpha) \cdot \cos(\psi_{AB}))$$

$$\delta_3(\psi_{AB},\alpha) := \delta_{t3}(\psi_{AB},\alpha) + \delta_{f3}(\psi_{AB},\alpha) \qquad \delta_3(\psi_{AB},\alpha_M) = 0.035\,mm$$

Déplacement angulaire de flexion $M_{tv}(\alpha,\alpha') := \sin(\alpha - \alpha')$ $\qquad M_{fv}(\alpha,\alpha') := -\cos(\alpha - \alpha')$

$$\theta_{ft}(\psi_{AB},\alpha) := \frac{F_z \cdot R^2}{2 \cdot G \cdot J_t} \cdot \left[-\sin(\psi_{AB}) \cdot \sin(\alpha) + 2 \cdot (1 - \cos(\alpha)) + \alpha \cdot \sin(\psi_{AB} - \alpha)\right]$$

$$\theta_{ff}(\psi_{AB},\alpha) := \frac{F_z \cdot R^2}{2 \cdot E \cdot I_{33}} \cdot (\sin(\psi_{AB} - \alpha) \cdot \alpha + \sin(\psi_{AB}) \cdot \sin(\alpha)) \qquad \theta_f(\psi_{AB},\alpha) := \theta_{ft}(\psi_{AB},\alpha) + \theta_{ff}(\psi_{AB},\alpha)$$

$\theta_{ff}(\psi_{AB},\alpha_M) = 0.193\,deg$ $\qquad \theta_{ft}(\psi_{AB},\alpha_M) = 0.061\,deg$ $\qquad \theta_f(\psi_{AB},\alpha_M) = 0.254\,deg$

Déplacement angulaire de torsion $M_{tv}(\alpha,\alpha') := \cos(\alpha - \alpha')$ $\qquad M_{fv}(\alpha,\alpha') := \sin(\alpha - \alpha')$

$$\theta_{tt}(\psi_{AB},\alpha) := \frac{F_z \cdot R^2}{2 \cdot G \cdot J_t} \cdot \left[-\alpha \cdot \cos(\psi_{AB} - \alpha) + \sin(\alpha) \cdot (2 - \cos(\psi_{AB}))\right]$$

$$\theta_{tf}(\psi_{AB},\alpha) := \frac{F_z \cdot R^2}{2 \cdot E \cdot I_{22}} \cdot (-\alpha \cdot \cos(\psi_{AB} - \alpha) + \sin(\alpha) \cdot \cos(\psi_{AB})) \qquad \theta_t(\psi_{AB},\alpha) := \theta_{tt}(\psi_{AB},\alpha) + \theta_{tf}(\psi_{AB},\alpha)$$

$\theta_{tf}(\psi_{AB},\alpha_M) = -0.077\,deg$ $\qquad \theta_{tt}(\psi_{AB},\alpha_M) = 0.135\,deg$ $\qquad \theta_t(\psi_{AB},\alpha_M) = 0.058\,deg$

Déplacements cartésiens en M par matrice de souplesse

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Arc de cercle E_L - Matrice S.mcd(R)

$$\boldsymbol{T} := \left[F_x \cdot N^{-1} \quad F_y \cdot N^{-1} \quad F_z \cdot N^{-1} \quad C_x \cdot (N \cdot m)^{-1} \quad C_y \cdot (N \cdot m)^{-1} \quad C_z \cdot (N \cdot m)^{-1}\right]^T$$

$\psi_F := \psi_{AB}$ $\qquad \Delta(\psi_F,\alpha) := \boldsymbol{S_F}(\psi_F,\alpha) \cdot \boldsymbol{T}$

$\delta_1(\psi_F,\alpha) := \Delta(\psi_F,\alpha)_1 \cdot m$ $\qquad \delta_2(\psi_F,\alpha) := \Delta(\psi_F,\alpha)_2 \cdot m$ $\qquad \delta_3(\psi_F,\alpha) := \Delta(\psi_F,\alpha)_3 \cdot m$

$\theta_1(\psi_F,\alpha) := \Delta(\psi_F,\alpha)_4$ $\qquad \theta_2(\psi_F,\alpha) := \Delta(\psi_F,\alpha)_5$ $\qquad \theta_3(\psi_F,\alpha) := \Delta(\psi_F,\alpha)_6$

$\delta_1(\psi_F,\alpha_M) = 0\,mm$ $\qquad \delta_2(\psi_F,\alpha_M) = 0\,mm$ $\qquad \delta_3(\psi_F,\alpha_M) = 0.035\,mm$

$\theta_1(\psi_F,\alpha_M) = 0.157\,deg$ $\qquad \theta_2(\psi_F,\alpha_M) = 0.208\,deg$ $\qquad \theta_3(\psi_F,\alpha_M) = 0\,deg$

Cas particuliers

Référence :E:\Résonateur (TA)\Fils et lames en arc de cercle\Définition Atan.mcd(R)

Quart de cercle

$\psi_{AB} := 90\cdot deg \qquad L := R\cdot\psi_{AB} \qquad L = 33.772\,mm$

$\delta_3(\psi_{AB},\psi_{AB}) = 0.177\,mm \qquad \theta_t(\psi_{AB},\psi_{AB}) = -0.192\,deg \qquad \theta_f(\psi_{AB},\psi_{AB}) = 0.435\,deg$

$$\Delta_{90} := F_z\cdot R^3\cdot\left[\frac{0}{N\cdot m^2} \quad \frac{0}{N\cdot m^2} \quad \left[\frac{\pi}{4}\cdot\frac{1}{E\cdot I_{22}} + \left(\frac{3\cdot\pi}{4}-2\right)\cdot\frac{1}{G\cdot J_t}\right]\right]^T \qquad \Delta_{90} = \begin{pmatrix}0\\0\\0.177\end{pmatrix} mm$$

$$\Theta_t := \frac{F_z\cdot R^2}{4}\cdot\left(\frac{-\pi}{E\cdot I_{22}} + \frac{4-\pi}{G\cdot J_t}\right) \qquad \Theta_t = -0.192\,deg \qquad \Theta_f := \frac{F_z\cdot R^2}{2}\cdot\left(\frac{1}{E\cdot I_{22}} + \frac{1}{G\cdot J_t}\right) \qquad \Theta_f = 0.435\,deg$$

Graphe de la déformation

$x_0(\alpha) := R\cdot cos(\alpha) \qquad y_0(\alpha) := R\cdot sin(\alpha) \qquad z_0(\alpha) := 0$

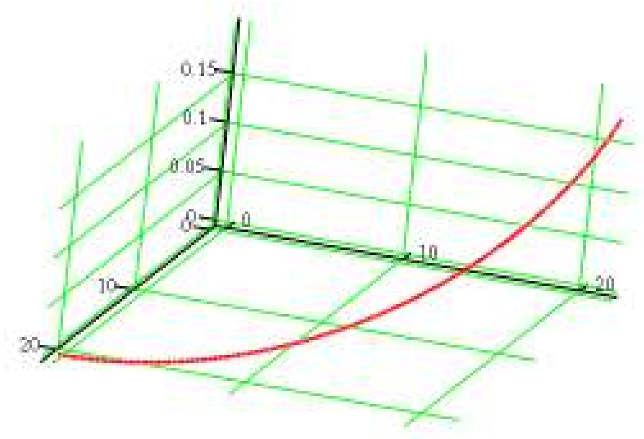
$$z_d(\alpha) := \delta_3(\psi_{AB},\alpha) \quad \theta_d(\alpha) := arctan\left(\frac{z_d(\alpha)}{R}\right) \quad x_d(\alpha) := R\cdot cos(\theta_d(\alpha))\cdot cos(\alpha) \quad y_d(\alpha) := R\cdot cos(\theta_d(\alpha))\cdot sin(\alpha)$$

$$x'_d(\alpha) := \frac{d}{d\alpha}x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha}y_d(\alpha) \qquad z'_d(\alpha) := \frac{d}{d\alpha}z_d(\alpha)$$

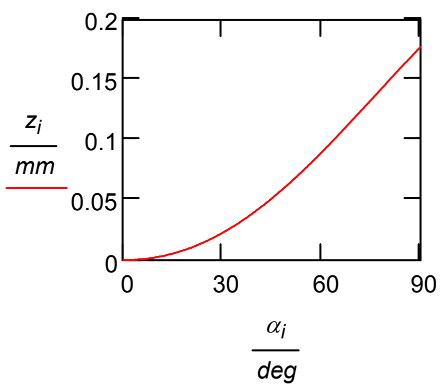
$L = 33.772\,mm$

$$\alpha_0 := 0 \qquad \alpha_{max} := \psi_{AB} \qquad L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2 + z'_d(\alpha)^2}\,d\alpha \qquad L_d = 33.772\,mm$$

$$n := 201 \qquad i := 1..n \qquad \alpha_i := \frac{\psi_{AB}}{n-1}\cdot(i-1) \qquad z_i := z_d(\alpha_i) \qquad x_i := x_d(\alpha_i) \qquad y_i := y_d(\alpha_i)$$



$\left(\frac{x}{mm}, \frac{y}{mm}, \frac{z}{mm}\right)$



Demi-cercle

$\psi_{AB} := 180 \cdot deg \qquad L := R \cdot \psi_{AB} \qquad L = 67.544\,mm$

$\delta_3(\psi_{AB}, \psi_{AB}) = 1.093\,mm \qquad \theta_t(\psi_{AB}, \psi_{AB}) = -1.367\,deg \qquad \theta_f(\psi_{AB}, \psi_{AB}) = 0.984\,deg$

$$\Delta_{180} := F_z \cdot R^3 \cdot \left[\frac{0}{N \cdot m^2} \quad \frac{0}{N \cdot m^2} \quad \left(\frac{\pi}{2} \cdot \frac{1}{E \cdot I_{22}} + \frac{3 \cdot \pi}{2} \cdot \frac{1}{G \cdot J_t} \right) \right]^T \qquad \Delta_{180} = \begin{pmatrix} 0 \\ 0 \\ 1.093 \end{pmatrix} mm$$

$$\Theta_t := \frac{F_z \cdot R^2}{2} \cdot \left(\frac{-\pi}{E \cdot I_{22}} + \frac{-\pi}{G \cdot J_t} \right) \qquad \Theta_t = -1.367\,deg \qquad \Theta_f := F_z \cdot R^2 \cdot \left(\frac{0}{E \cdot I_{22}} + \frac{2}{G \cdot J_t} \right) \qquad \Theta_f = 0.984\,deg$$

Graphe de la déformation

$x_0(\alpha) := R \cdot cos(\alpha) \qquad y_0(\alpha) := R \cdot sin(\alpha) \qquad z_0(\alpha) := 0$

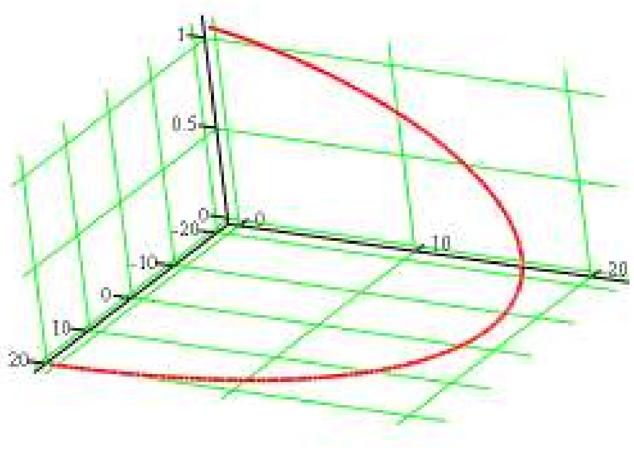
$$z_d(\alpha) := \delta_3(\psi_{AB}, \alpha) \quad \theta_d(\alpha) := arctan\left(\frac{z_d(\alpha)}{R} \right) \quad x_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot cos(\alpha) \quad y_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot sin(\alpha)$$

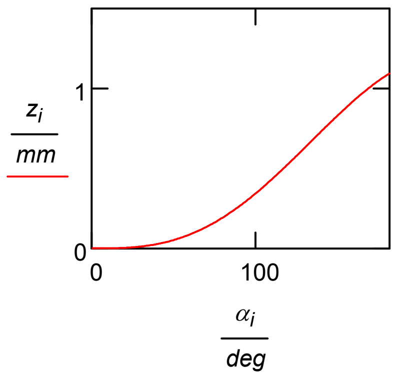
$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \qquad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$L = 67.544\,mm$

$$\alpha_0 := 0 \qquad \alpha_{max} := \psi_{AB} \qquad L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2 + z'_d(\alpha)^2}\, d\alpha \qquad L_d = 67.536\,mm$$

$$n := 201 \qquad i := 1 .. n \qquad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1) \qquad z_i := z_d(\alpha_i) \qquad x_i := x_d(\alpha_i) \qquad y_i := y_d(\alpha_i)$$





$$\left(\frac{x}{mm}, \frac{y}{mm}, \frac{z}{mm} \right)$$

Anneau fendu

$\psi_{AB} := 360 \cdot deg \qquad L := R \cdot \psi_{AB} \qquad L = 135.088\, mm$

$\delta_3(\psi_{AB}, \psi_{AB}) = 2.186\, mm \qquad \theta_t(\psi_{AB}, \psi_{AB}) = -2.735\, deg \qquad \theta_f(\psi_{AB}, \psi_{AB}) = 0\, deg$

$$\Delta_{360} := F_z \cdot R^3 \cdot \left[\frac{0}{N \cdot m^2} \quad \frac{0}{N \cdot m^2} \quad \left(\pi \cdot \frac{1}{E \cdot I_{22}} + 3 \cdot \pi \cdot \frac{1}{G \cdot J_t} \right) \right]^T \qquad \Delta_{360} = \begin{pmatrix} 0 \\ 0 \\ 2.186 \end{pmatrix} mm$$

$$\Theta_t := \frac{F_z \cdot R^2}{2} \cdot \left(\frac{-2 \cdot \pi}{E \cdot I_{22}} + \frac{-2 \cdot \pi}{G \cdot J_t} \right) \qquad \Theta_t = -2.735\, deg \qquad \Theta_f := F_z \cdot R^2 \cdot \left(\frac{0}{E \cdot I_{22}} + \frac{0}{G \cdot J_t} \right) \qquad \Theta_f = 0\, deg$$

Graphe de la déformation

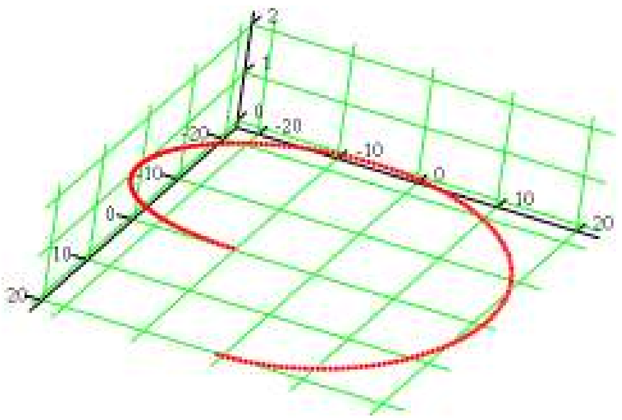
$x_0(\alpha) := R \cdot cos(\alpha) \qquad y_0(\alpha) := R \cdot sin(\alpha) \qquad z_0(\alpha) := 0$

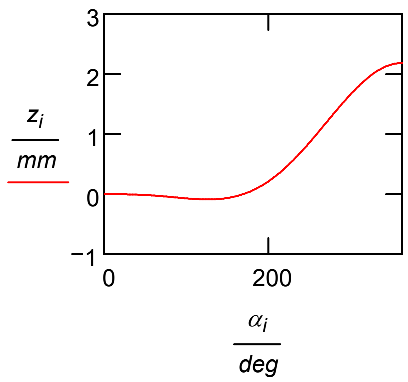
$$z_d(\alpha) := \delta_3(\psi_{AB}, \alpha) \quad \theta_d(\alpha) := arctan\left(\frac{z_d(\alpha)}{R} \right) \quad x_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot cos(\alpha) \quad y_d(\alpha) := R \cdot cos(\theta_d(\alpha)) \cdot sin(\alpha)$$

$$x'_d(\alpha) := \frac{d}{d\alpha} x_d(\alpha) \qquad y'_d(\alpha) := \frac{d}{d\alpha} y_d(\alpha) \qquad z'_d(\alpha) := \frac{d}{d\alpha} z_d(\alpha)$$

$$\alpha_0 := 0 \qquad \alpha_{max} := \psi_{AB} \qquad L_d := \int_{\alpha_0}^{\alpha_{max}} \sqrt{x'_d(\alpha)^2 + y'_d(\alpha)^2 + z'_d(\alpha)^2}\, d\alpha \qquad \begin{matrix} L = 135.088\, mm \\ L_d = 134.987\, mm \end{matrix}$$

$$n := 201 \qquad i := 1 .. n \qquad \alpha_i := \frac{\psi_{AB}}{n-1} \cdot (i-1) \qquad z_i := z_d(\alpha_i) \qquad x_i := x_d(\alpha_i) \qquad y_i := y_d(\alpha_i)$$





$$\left(\frac{x}{mm}, \frac{y}{mm}, \frac{z}{mm} \right)$$